CRANBROOK SCHOOL

Year 12 (2U) Mathematics

HSC Trial Examination

Friday July 23, 2010

Time Allowed: 3 hours *plus* 5 minutes reading time

Total Marks: 120

There are 10 questions, each of equal value.
Start a new booklet for each question.
All necessary working should be shown in every question.
Full marks may not be awarded if work is careless or badly arranged.
Approved calculators may be used.
A page of standard integrals appears on the back page of this booklet

Question 1(12 Marks) **Start a new Answer Booklet** **Marked by**

(a) Evaluate $\sqrt{\dfrac{3.2\times 5.6}{5.4^2}}$ correct to 3 significant figures **2**

(b) Solve the equation $|2x-1|\geq 3$ **2**

(c) Find the exact value of $\sec\dfrac{\pi}{3}$ **2**

(d) Find the integers a and b such that $\dfrac{2}{2-\sqrt{3}}=a+b\sqrt{3}$ **2**

(e) Draw a neat sketch of the function $y=\sqrt{9-x^2}$, showing all intercepts **2**

(f) Factorise fully $48x-3x^3$ **2**

Question 2 (12 Marks) **Start a new Answer Booklet** **Marked by**

(a) Differentiate with respect to x:

i. $(2x+1)^3$ 1

ii. $\dfrac{\sin x}{e^x}$ 2

iii. $x\log_e x$ 2

(b) Find

i. $\int \sqrt{x}\,dx$ 1

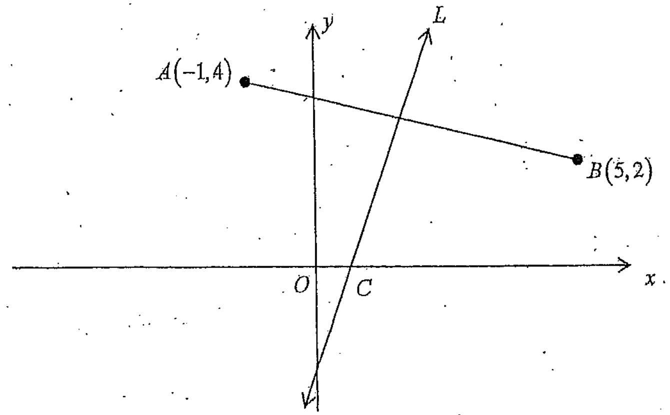
ii. $\int \cos x\,dx$ 1

iii. $\int \dfrac{x+2}{x^2+4x}\,dx$ 2

(c) Evaluate $\int_0^{\frac{\pi}{2}} \sin 2x\,dx$ 3

Question 3 (12 Marks) **Start a new Answer Booklet** **Marked by**

(a) The diagram below shows the points $A(-1,4)$ and $B(5,2)$. The line L has equation $3x-y-3=0$ and cuts the x-axis at C.



i. Show that the length of AB is $2\sqrt{10}$ units 1

ii. Find the coordinates of M, the midpoint of AB 1

iii. Find the gradient of AB 1

iv. Show that the equation of AB is $x+3y-11=0$ 1

v. Prove that L is the perpendicular bisector of AB 2

vi. Find the coordinates of C 1

vii. Write down the equation of the circle with AB as the diameter. 1

(b) α and β are the roots of the equation $x^2-6x+10=0$. Find the values of:

i. $\alpha+\beta$ 1

ii. $\alpha\beta$ 1

iii. $(\alpha+1)(\beta+1)$ 2

Question 4 (12 Marks) **Start a new Answer Booklet** **Marked by**

(a) Consider the curve $y = x^3 - Ax^2 + 9x + 4$.

i. Find the value of A given that it has a maximum turning point at (1, 8) **1**

ii. Find an expression for $\frac{dy}{dx}$ **1**

iii. Hence find the other turning point and determine its nature **2**

iv. Draw a neat sketch of the curve, labelling all features **2**

(b) The equation of a parabola is given by $(x+2)^2 = 8(y-1)$

i. Find the focal length **1**

ii. Find the coordinates of the vertex **1**

iii. Find the coordinates of the focus **1**

iv. Write down the equation of the directrix **1**

v. Draw a neat sketch showing all essential features. **2**

Question 5 (12 Marks) **Start a new Answer Booklet** **Marked by**

(a) Solve the equation $3^{2x} - 10(3^x) + 9 = 0$ **3**

(b) Use Simpsons rule with 7 function values to find the exact value of $\int_0^6 \sqrt{e^x}\, dx$ **3**

(c) The population P of a penguin colony is growing at a rate that is proportional to the current population. The population at any time t years is given by:

$$P = P_0 e^{kt}$$

Where P_0 and k are constants

The population at time $t = 0$ was 2000 and at time $t = 2$ was 6000

i. Find the value of P_0 **1**

ii. Find the value of k in exact form. **1**

iii. At what time, correct to 1 decimal place, will the population reach 12000? **2**

iv. What will the population be after 10 years? **1**

v. Draw a neat graph to illustrate the population over time. **1**

Question 6 (12 Marks) **Start a new Answer Booklet** **Marked by**

(a) Alex walks 8 km on a bearing of 140°T.

She then turns and walks on a bearing of 060°T for 2 km.

i. Draw a diagram to illustrate the problem. **1**

If Alex wants to return to her starting point, calculate:

ii. The shortest distance she will need to travel correct to 1 decimal place. **2**

iii. The new bearing she will need to walk on to get back to her starting point correct to the nearest minute. **2**

(b)

i. Simplify $\log_2 4$ **1**

ii. Hence solve the equation $\log_2(x+1)-\log_2 x=2$ **2**

(c) The graphs of $y=2x$ and $y=6x-x^2$ intersect at the origin and the point B.

i. Illustrate with a neat sketch. **1**

ii. Find the coordinates of B **1**

iii. Calculate the area between the two graphs. **2**

Question 7 (12 Marks) **Start a new Answer Booklet** **Marked by**

(a)

i. Draw a neat sketch of $y=\sin x$ and $y=\sqrt{3}\cos x$ on the same axes in the domain $-\pi \le x \le \pi$ **2**

ii. Hence state how many solutions the equation $\sin x=\sqrt{3}\cos x$ has in the domain $-\pi \le x \le \pi$ **1**

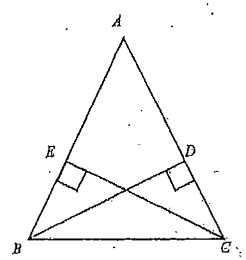
iii. Solve the equation $\sin x=\sqrt{3}\cos x$ *algebraically* for $-\pi \le x \le \pi$ **2**

(b)

i. On the same set of axes draw a neat sketch of $x^2+y^2=9$ and $y=|x|$ clearly showing the x and y intercepts **2**

ii. Shade the region defined by $x^2+y^2 \le 9$, $y \ge |x|$ and $x \ge 0$ **2**

(c) In ΔABC, altitudes BD and CE are equal. Prove $\Delta BDA \equiv \Delta CEA$ **3**

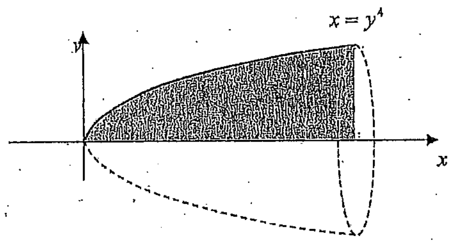


Question 8 (12 Marks) **Start a new Answer Booklet** **Marked by**

(a) Find the equation of the tangent to the curve $y = x\cos x$ at $x = \frac{\pi}{2}$ 3

(b) When the area of the shaded region is rotated around the $x-$axis it forms the tip of a bullet.

If the length of the bullet is measured to be 1 cm long, calculate its volume. 3



(c) At time t seconds, the position of a point moving in a straight line along the x-axis is given by $x = at^2 + bt$ where a and b are constants.

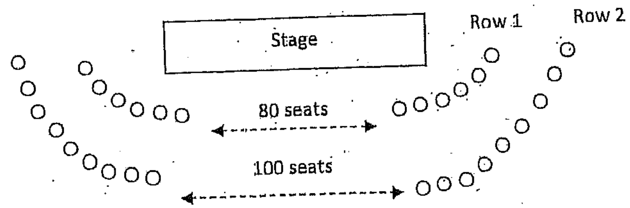
If it passes through O, the origin, with velocity 24 cm/s in the positive direction at time $t = 0$, and after 8 seconds it is again at O, find the values of a and b. 3

(d) For a particle moving along the x-axis, the acceleration is given by $a = e^{2t} + \frac{1}{e^{2t}}$ where x and t are measured in centimetres and seconds respectively.

If the particle is initially at rest at the origin, find the exact position of the particle when $t = 3$ 3

Question 9 (12 Marks) **Start a new Answer Booklet** **Marked by**

(a) There are 80 seats in the first row of a concert hall. The next row has 100 and so on such that the next row has 20 more seats than the previous row.



Calculate:

i. Find the nth term rule that will find how many seats are in each row. 1

ii. How many seats are there in the 10^{th} row? 1

iii. If there 11100 seats in the concert hall, how many rows are there? 2

(b) Hanna wishes to buy a used car that is on sale for \$12 000. The car salesman offers her the following deal. The first 3 months she will not have to pay any interest but the first repayment of \$$M$ will be due at the end of the first month. Interest will be charged at 1% a month, calculated before the monthly repayment is made. If Hanna plans to pay back the loan in 6 years;

i. Find an expression for A_3. 1

ii. Show that $A_5 = (12000 - 3M)(1.01)^2 - M(1 + 1.01)$ 2

iii. Hence write down an expression for A_{72} 1

iv. Find the value of M 2

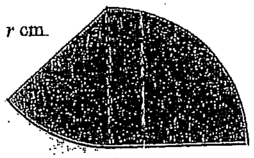
(c) A series is defined as $(\sqrt{2}-1) + (\sqrt{2}-1)^2 + (\sqrt{2}-1)^3 + \ldots$

Find its limiting sum. 2

Question 10: (12 Marks) **Start a new Answer Booklet** **Marked by**

(a) Part of a locking mechanism is shown below. It is made up of a quarter circle with radius r cm and a sector with radius r cm and angle θ radians at the centre.

r cm



r cm

(i) Show that the area of the locking mechanism is given by: **1**

$$A = \frac{1}{2}r^2\left(\theta + \frac{\pi}{2}\right) \text{ cm}^2$$

(ii) If the area of the locking mechanism is 1 cm^2 show that: **1**

$$\theta = \frac{2}{r^2} - \frac{\pi}{2} \text{ radians.}$$

(iii) Show that the perimeter of the locking mechanism is given by: **2**

$$P = 2r + r\theta + \frac{\pi}{2}r \text{ cm.}$$

(iv) Show that the least perimeter occurs when $r = 1$ cm. **1**

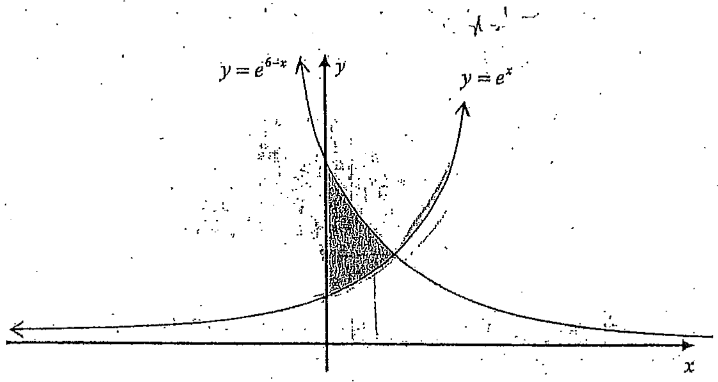
(v) Hence or otherwise find the value of θ *in degrees* that gives a minimum perimeter. **1**

(b) Given that $\frac{d}{dy}(y\ln y - y) = \ln y$ it follows that $\int \ln y \, dy = y\ln y - y + c$

Show that $\frac{d}{dy}(y\ln y - 7y) = \ln y - 6$ **2**

Question 10 continues over the page

(c) The following diagram shows an area bound by the curves $y = e^x$ and $y = e^{6-x}$ and the y-axis.



i. Show the point of intersection of the curves is $(3, e^3)$ **1**

ii. Hence, *using the results from part (b)* calculate the area of the shaded region. **3**

End of Examination

2u TRIAL HSC 2010 ✓ = 1 mark

Q 1 a) 0·784 (3 sf) ✓✓ Very well done

b) $|2x-1| \geq 3$ quite well done

$2x-1 \leq -3$ $\quad$ $2x-1 \geq 3$

$2x \leq -2$ $\quad$ $2x \geq 4$

$x \leq -1$ $\quad$ $x \geq 2$ ✓✓

c) $\sec\frac{\pi}{3} = \frac{1}{\cos\frac{\pi}{3}}$ ✓✓

$= 2$

d) $\frac{2}{2-\sqrt{3}} = a + b\sqrt{3}$

$LHS = \frac{2}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}}$

$= \frac{4+2\sqrt{3}}{1}$ ✓✓

$\therefore a = 4$

$b = 2$

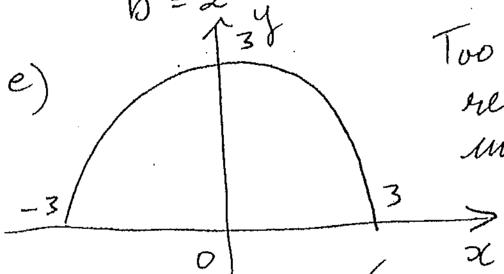
f) $48x - 3x^3$

$= 3x(16-x^2)$ ✓✓

$= 3x(4-x)(4+x)$

good but a couple of students decided it equalled zero ... NO!

e)



✓✓

Too many did not recognise the semi-circle more revision on basic curves is needed.

also label axes.

Question 2: (CRA)

(a)

(ii) - Many students differentiated $\sin x$ to $-\cos x$.

- Students needed to factorise their answer and simplify their answer to get full marks.

(iii) - Most students were able to differentiate correctly. In all cases more working needed to be shown. This is not a one step problem.

(b) (i) - Students were helped by writing $\sqrt{x}$ as $x^{\frac{1}{2}}$

(ii) - Students need to remember "+ C" to achieve full marks. If a student forgot "+ C" in all of part (b), only 1 mark was deducted.

(iii) - Students need to demonstrate lines of working. Many students were confused whether to multiply by 2 or $\frac{1}{2}$.

(c) - Students were awarded one mark for correctly integrating.

- One mark was given for correctly subbing in $\pi/2$ and 0, and simplifying the cos.

- The third mark was given for a correct answer.

- e.c.f. (error carried forward) was given if students incorrectly integrated but completed the following steps successfully. One mark was awarded in this scenario.

Question 2:

(a)

(i) $(2x+1)^3$

$= 3(2x+1)^2 \times 2$

$= 6(2x+1)^2$

(ii) $\dfrac{\sin x}{e^x} = \dfrac{e^x \cos x - e^x \sin x}{(e^x)^2}$

$= \dfrac{e^x(\cos x - \sin x)}{(e^x)^2}$

$= \dfrac{\cos x - \sin x}{e^x}$

(iii) $x \log_e x$

$= \left(x \times \frac{1}{x}\right) + (\log_e x \times 1)$

$= 1 + \ln x.$

(b)

(i) $\int \sqrt{x}\, dx$

$= \int x^{1/2}\, dx$

$= \dfrac{2x^{3/2}}{3} + c$

(ii) $\int \cos x\, dx$

$= \sin x + c$

(iii) $\int \dfrac{x+2}{x^2+4x}\, dx$

$= \ln(x^2+4x) \times \frac{1}{2} + c$

$= \dfrac{\ln(x^2+4x)}{2} + c$

(c) $\int_0^{\pi/2} \sin 2x\, dx$

$= \left[-\frac{1}{2}\cos 2x\right]_0^{\pi/2}$

$= -\frac{1}{2}\left[\cos \pi - \cos 0\right]$

$= -\frac{1}{2}(-1-1)$

$= 1$

Question 3: Solns + Marker's Notes

(a) (i) $d_{AB} = \sqrt{(5+1)^2 + (2-4)^2}$

$= \sqrt{36+4}$

$= \sqrt{40}$

$= 2\sqrt{10}$ units ✓

(ii) $M_{AB} = \left(\frac{5-1}{2}, \frac{2+4}{2}\right)$

$= (2,3)$ ✓

(iii) $m_{AB} = \dfrac{2-4}{5+1}$

$= -\frac{1}{3}$ ✓

(iv) $y-2 = -\frac{1}{3}(x-5)$

$3y - 6 = -x + 5$

$x + 3y - 11 = 0$ ✓

(v) • Sub $(2,3)$ into l

• Check $m_l \times -\frac{1}{3} = -1$

$3(2) - 3 - 3 = 0$

$0 = 0$

$\therefore l$ passes thru $(2,3)$ ✓

m_l: $3x - y - 3 = 0$

$y = 3x - 3$

$m_l = 3$

$3 \times -\frac{1}{3} = -1$ $\therefore$ perpendicular ✓

(vi) C is x-int of l

$\therefore 3x - 0 - 3 = 0$

$3x = 3$

$x = 1$

$\therefore C(1,0)$ ✓

(vii) Radius $= \frac{1}{2}(2\sqrt{10})$

centre $(2,3)$

$\therefore$ eqn: $(x-2)^2 + (y-3)^2 = 10$ ✓

(b) (i) $\alpha + \beta = \dfrac{-b}{a} = 6$ ✓

(ii) $\alpha\beta = \dfrac{c}{a} = 10$ ✓

(iii) $(\alpha+1)(\beta+1)$

$= \alpha\beta + \alpha + \beta + 1$ ✓

$= 10 + 6 + 1$

$= 17$ ✓

Question 3 Marker's Notes:

- Generally well done however the most commonly lost marks were in (a)(vi) where students just gave the x-value and not the coordinates of C.
Also, major mix-ups in (a)(vii) where some students failed to recall the eqn of a circle!!

Question 4: Solns. + Marker's Notes.

(a) $y = x^3 - Ax^2 + 9x + 4$

(i) $8 = 1^3 - A(1)^2 + 9(1) + 4$
$8 = 14 - A$
$A = 6$ ✓

• well done

(ii) $\frac{dy}{dx} = 3x^2 - 12x + 9$

• Marks were still awarded if answer was given as $3x^2 - 2Ax + 9$.

(iii) $3x^2 - 12x + 9 = 0$
$x^2 - 4x + 3 = 0$
$(x-3)(x-1) = 0$
$x = 1, 3$

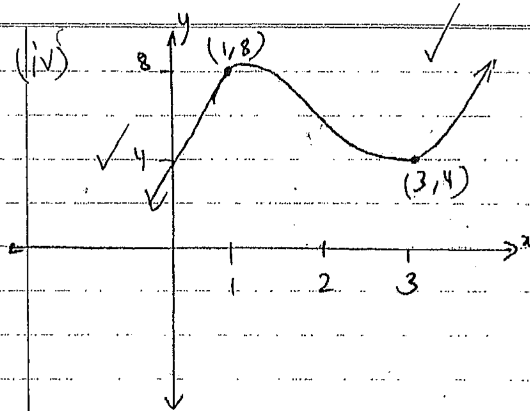
• well done

$\therefore$ SP at $(1,8)$ and $(3,4)$ ✓

$\frac{d^2y}{dx^2} = 6x - 12$

when $x = 3$, $\frac{d^2y}{dx^2} = 6 > 0$ ✓
$\therefore$ MIN

(iv)



• Marks were given for neat scale/axis and showing y-intercept
• No marks were deducted for not finding the point of inflexion.

(b) (i) $a = 2$ ✓

(ii) $V(-2, 1)$ ✓

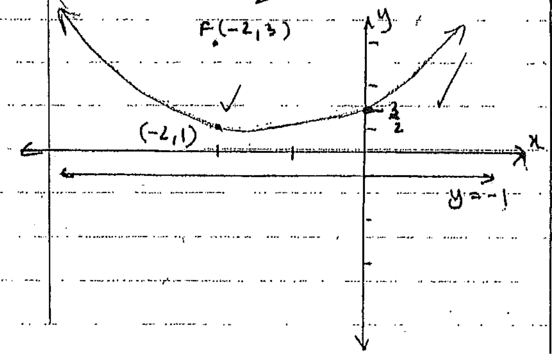
(iii) $F(-2, 3)$ ✓

(iv) $y = -1$ ✓

Mostly well done. A large number of students did not even attempt this qn!? The preliminary course is assumed knowledge and will be examined in the HSC!

(v)

y-int: $4 = 8y - 8$
$12 = 8y$
$y = \frac{3}{2}$



• Marks were awarded for neatness, labels and intercept of y-axis.

Question 5: (CRA)

(a) - Many students tried to solve the equation without letting $3^x = u$ (or another value) - one mark was given for letting $3^x = \_\_$.
- One mark was given for correctly finding values for 3^x
- One mark was given for correctly finding x.

(b) - This question asked for the exact value!
- Marks were awarded for finding the 7 function values, finding "h" in the Simpsons rule equation and correctly using the function values in the equation.

(c) (ii) k needed to be in exact form!

(iii) Students could use the value for "k" that they found in part (ii) - e.c.f was applied.

(iv) Again, see part (iii)

(v) One mark was awarded if students indicated that population started at 2000 when t=0, and population was 486,000 (or what they found in part(iv)) at t=10.

Question 5:

(a) $3^{2x} - 10(3^x) + 9 = 0$

let $u = 3^x$

$\therefore u^2 - 10u + 9 = 0$

$(u-9)(u-1) = 0$

$\therefore u = 9, 1$

$\therefore 3^x = 9$ and $3^x = 1$

$\therefore x = 2, 0$

(b) $\int_0^6 \sqrt{e^x}\,dx$

$\therefore h = 1$

0	1	2	3	4	5	6
1	$\sqrt{e}$	$\sqrt{e^2}$	$\sqrt{e^3}$	$\sqrt{e^4}$	$\sqrt{e^5}$	$\sqrt{e^6}$
	$\times 4$	$\times 2$	$\times 4$	$\times 2$	$\times 4$	

$\therefore A = \frac{1}{3}\left[1 + 4\left(\sqrt{e} + \sqrt{e^3} + \sqrt{e^5}\right) + 2\left(\sqrt{e^2} + \sqrt{e^4}\right)\right]$

$\therefore A = \frac{1}{3}\left[1 + 4\left(\sqrt{e} + \sqrt{e^3} + \sqrt{e^5}\right) + 2\left(e + e^2\right)\right]$ (exact value)

(c)

(i) $P_0 e^0 = 2000$

$\therefore P_0 = 2000$

(ii) $P = 2000e^{kt}$ $\quad P = 6000, t = 2$

$\therefore 6000 = 2000e^{2k}$

$3 = e^{2k}$

$\ln 3 = 2k$

$\therefore k = \frac{\ln 3}{2}$

(c)

(iii) $12000 = 2000 e^{kt}$

$6 = e^{kt}$

$\ln 6 = \ln e^{kt}$

$\ln 6 = e^{kt}$

since $k = \frac{\ln 3}{2}$

$\therefore \ln 6 = \frac{\ln 3}{2} \times t$

$\therefore t = \frac{2 \ln 6}{\ln 3}$

$\therefore t = 3.3$ years.

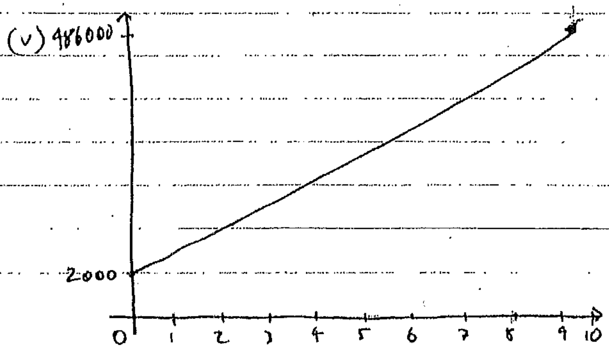
(iv) $P = 2000 e^{10k}$

$= 2000 \times e^{10 \times \frac{\ln 3}{2}}$

$= 2000 \times 243$

$= 486000$ penguins.

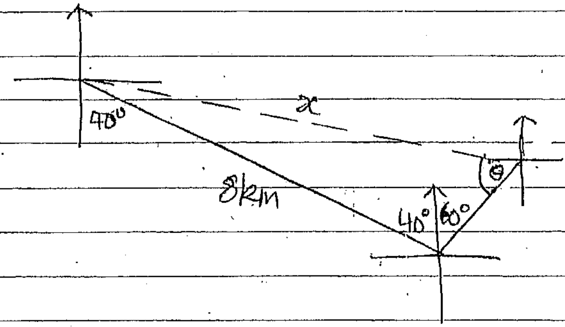
(v)



QUESTION 6.

(a)

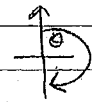
(i)



✓

- Mostly answered correctly.
- A few students forgot that bearings are measured from north in a clockwise direction

eg



(ii) $x^2 = 8^2 + 2^2 - 2.8.2 \cos 100°$ ✓

$x^2 = 73.5 \ldots$

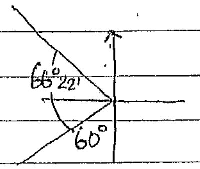
$\therefore x = 8.6$ km ✓

- Mostly correct
- Some students assumed the triangle was right-angled.

(iii) $\frac{\sin 100°}{8.6} = \frac{\sin \theta}{8}$

$\theta = \sin^{-1}\left(\frac{8 \sin 100°}{8.6}\right)$

$= 66.4° = 66°22'$ ✓



$\therefore$ Bearing $= 180° + 60° + 66°22'$
$= 306°22'$ ✓

- Could have been answered better
- A lot of students forgot to use the sine rule.
- Many also forgot to find the bearing needed.

(b)

(i) $\log_2 4 = \log_2 2^2$

$= 2\log_2 2 = 2$ ✓

(ii) $\log_2(x+1) - \log_2 x = 2$

$\log_2\left(\frac{x+1}{x}\right) = \log_2 4$

$\frac{x+1}{x} = 4$ ✓

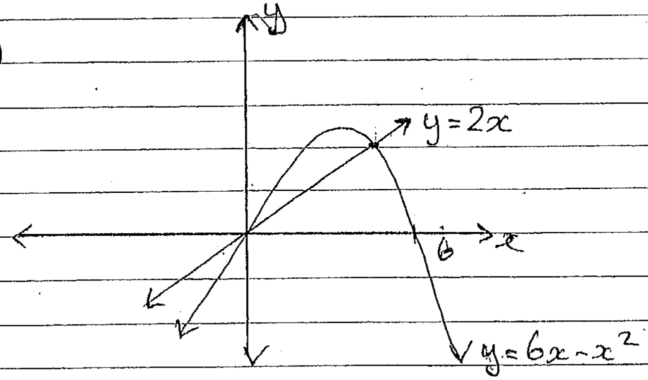
$x+1 = 4x$

$1 = 3x$

$\therefore x = \frac{1}{3}$ ✓

(c)

(i)



✓

- surprisingly answered poorly
- many students left off the x-intercepts, which are fundamental for such a basic sketch.
- other students didn't recognise a concave-down parabola from its equation.

(ii) $2x = 6x - x^2$

$x^2 - 4x = 0$

$x(x-4) = 0$

$\therefore x = 0, 4 \Rightarrow (4, 8)$ ✓

- most answered correctly. Well done!

(iii) $\int_0^4 \underbrace{6x - x^2}_{\text{top f'n}} \underbrace{- 2x}_{\text{bottom f'n}} \, dx$

$= \int_0^4 4x - x^2 \, dx$ ✓

$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$

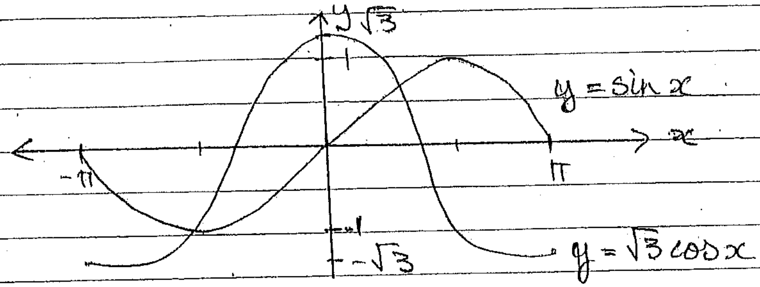
$= \left(32 - \frac{64}{3}\right) - 0$

$= 10\frac{2}{3}$ units^2 or $\frac{32}{3}$ units^2 ✓

- mostly correct
- failure to identify the "top function" or using incorrect limits were the main source of errors.

QUESTION 7.

(a)
(i)



$y = \sin x$ correct ✓ $y = \sqrt{3}\cos x$ correct ✓

- the cohort should have answered better.
- many students confused the domain of the f'ns.
- some made silly errors such as not even including the axes.

(ii) 2 (from the graph) ✓

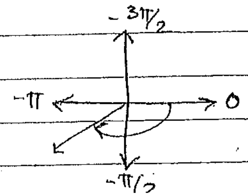
- most answered correctly.
- a few students didn't see the word "hence" and found the answer by other methods.

(iii) $\sin x = \sqrt{3}\cos x$

$\frac{\sin x}{\cos x} = \sqrt{3}$

$\tan x = \sqrt{3}$

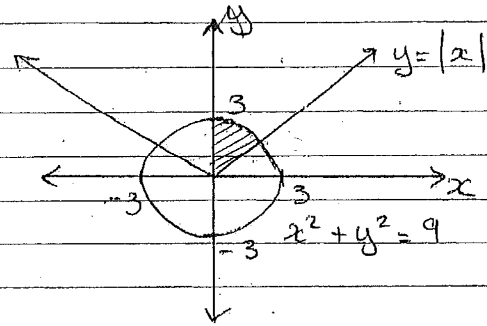
$\therefore x = \pi/3$ in Q1 ✓



In negative quadrant $x = -\pi + \pi/3$

$= -\frac{2\pi}{3}$ ✓

(bi)
(i)



- Mostly correct ... the absolute value f'n proved to be main source of errors.

(ii) see above

- Could have been answered better
- a lot of students neglected the $x \geq 0$ constraint

(c) In ΔBDA & ΔCEA ← ALWAYS SPECIFY WHICH Δs.

$\angle CEA = \angle BDA = 90°$ ($\angle$s on a straight line add to 180°) ✓

$BD = EC$ (given) ✓ ← MAKE REASONS CONCISE.

$\angle BAC$ is common ✓

SPECIFY EACH ARM OF THE $\angle$

$\Delta BDA \equiv \Delta CEA$ (AAS) ✓

SPECIFY WHICH TEST WAS USED

- Very poorly answered
- Very few students managed to correctly us a clear & concise 5 line proof.
- A lot actually proved that different triangles were congruent.
- Some used RHS test but did not prove that the hypotenuses were equal.
- If you find that you are writing a novel, STOP, step back & rethink your strategy!

QUESTION 8

(a) If $x = \pi/2$, $y = \pi/2 \cos \pi/2$
$= 0$

$\therefore$ co-ordinates are $(\pi/2, 0)$ ✓

If $y = x \cos x$

$u = x \quad v = \cos x$

$u' = 1 \quad v' = -\sin x$

$y' = vu' + uv'$
$= \cos x - x \sin x$ ✓

At $x = \pi/2$, $m = \cos \pi/2 - \pi/2 \sin \pi/2$
$= 0 - \pi/2$
$= -\pi/2$

$y - 0 = -\pi/2 (x - \pi/2)$

$y = -\pi/2\, x + \pi^2/4$ ✓

or

$2\pi x + 4y - \pi^2 = 0$

- Could have been answered much better.
- A few students couldn't identify the need for the product rule.

* When giving the equation of a straight line, it must be either in general form or gradient-intercept form.

(b) If $x = y^4$ (If $x = 1$, $y = 1$)
$\sqrt{x} = y^2$ ✓

$V = \pi \int_0^1 y^2 \, dx = \pi \int_0^1 \sqrt{x} \, dx$

$= \pi \left[\frac{2x^{3/2}}{3} \right]_0^1$ ✓

$= \pi \left(2/3 - 0 \right)$

$= 2\pi/3$ units^3 ✓

- mostly correct.
- finding y^2 proved to be the main stumbling block

(c) $x = at^2 + bt$

$v = \frac{dx}{dt} = 2at + b$ ✓

$\left.\begin{array}{l} v = 24 \\ t = 0 \end{array}\right\}$ $24 = 0 + b$ $\quad b = 24$ ✓

$\therefore x = at^2 + 24t$

$\left.\begin{array}{l} x = 0 \\ t = 8 \end{array}\right\}$ $0 = a.64 + 24.8$
$= 64a + 128 \quad \therefore a = -3$ ✓

- Mostly correct.
- Main source of errors were confusing integrating & differentiating.

(d) $a = e^{2t} + \frac{1}{e^{2t}} = e^{2t} + e^{-2t}$

$$V = \int e^{2t} + e^{-2t}\, dt$$

$$= \frac{e^{2t}}{2} - \frac{e^{-2t}}{2} + c_1$$

$\left.\begin{array}{l} V=0 \\ t=0 \end{array}\right\}$ $V = \frac{1}{2} - \frac{1}{2} + c_1$

$\therefore c_1 = 0$

$$V = \frac{e^{2t}}{2} - \frac{e^{-2t}}{2} \quad ✓$$

$$x = \int \frac{e^{2t}}{2} - \frac{e^{-2t}}{2}\, dt$$

$$= \frac{e^{2t}}{4} + \frac{e^{-2t}}{4} + c_2$$

$\left.\begin{array}{l} x=0 \\ t=0 \end{array}\right\}$ $x = \frac{1}{4} + \frac{1}{4} + c_2$

$\therefore c_2 = -\frac{1}{2}$

$$x = \frac{e^{2t}}{4} + \frac{e^{-2t}}{4} - \frac{1}{2} \quad ✓$$

At $t=3$ $x = \frac{e^6}{4} + \frac{e^{-6}}{4} - \frac{1}{2}$

or

$x = \frac{1}{4}\left(e^6 + e^{-6} - 2\right)$ ✓

or

$x = \frac{(e^6 - 1)^2}{4e^6}$ units

- Answered quite poorly.
- There were a lot of problems with correctly integrating e^{-2t}

* Importantly, many students neglected to use the correct symbols for integrating. It is important and may cost you marks in the HSC.

$$\int f(x)\, dx$$

integral symbol

function

which variable you are integrating with respect to!

QUESTION 1.

(a) (i) $T_1 = 80 = a$

$T_2 = 100$

$\therefore d = 20$

$\therefore T_n = a + (n-1)d$

$= 80 + (n-1) \times 20$

$= 60 + 20n$ (1)

(ii) $T_{10} = 60 + 200 = 260$ (1)

(iii) $S_n = 11100 = \frac{n}{2}(2a + (n-1)d)$ (1)

$= \frac{n}{2}(160 + (n-1) \times 20)$

Remembering this formula seems to have stumped some students

$\therefore 22200 = n(140 + 20n)$

$1110 = n(7 + n)$

$0 = n^2 + 7n - 1110$

$= (n+37)(n-30)$

$\therefore n = 30, -37.$

Recognising what to do with a quadratic is a crucial skill at this level!

$\therefore$ There are 30 rows in the hall (2)

(b) (i) $A_0 = 12000$

$A_1 = 12000 - M$

$A_3 = 12000 - 3M$ (1)

(ii) $A_4 = (12000 - 3M) \times 1.01 - M$ (1)

$A_5 = \left[(12000 - 3M) \times 1.01 - M\right] \times 1.01 - M$

$= (12000 - 3M) \times 1.01^2 - M \times (1.01 + 1)$ (2)

(iii) $A_{72} = (12000 - 3M) \times 1.01^{69} - M(1.01^{68} + 1.01^{67} + \dots + 1.01 + 1)$ (1)

(iv) Since $A_{72} = 0 = (12000 - 3M) \times 1.01^{69} - M\left(\frac{1.01^{69} - 1}{1.01 - 1}\right)$

Factorising out M so as to make it the subject of the equation was a challenge for many.

$M\left(\frac{1.01^{69} - 1}{0.01} + 3 \times 1.01^{69}\right) = 12000 \times 1.01^{69}$

$M = \dfrac{12000 \times 1.01^{69}}{\frac{1.01^{69} - 1}{0.01} + 3 \times 1.01^{69}}$

$= \$227.83$ (2)

(c) $a = \sqrt{2} - 1$

$r = \sqrt{2} - 1$

$|r| = |\sqrt{2} - 1| < 1$ $\therefore$ Limiting Sum exists (1)

very few performed this step, but didn't lose marks.

$S_\infty = \frac{a}{1-r} = \frac{\sqrt{2} - 1}{1 - \sqrt{2} + 1} = \frac{\sqrt{2} - 1}{2 - \sqrt{2}} = \frac{(\sqrt{2} - 1)(\sqrt{2} + 2)}{2}$

careful with $\left[1 - (\sqrt{2} - 1) \neq 1 - \sqrt{2} - 1\right]$!

$= \frac{2 - \sqrt{2} + 2\sqrt{2} - 2}{2}$

$= \frac{\sqrt{2}}{2}$ (2)

10.

✓ = 1 mark

i) A = Sector area + $\frac{1}{4}$ of a circle

$= \frac{\theta}{2}r^2 + \frac{1}{4}\pi r^2$ cm^2

$= \frac{1}{2}r^2(\theta + \frac{\pi}{2})$ cm^2 ✓

V Well done

ii) $\frac{1}{2}r^2(\theta + \frac{\pi}{2}) = 1$

$\theta + \frac{\pi}{2} = \frac{2}{r^2}$

$\theta = \frac{2}{r^2} - \frac{\pi}{2}$ radians ✓

(iii) $P = r + r +$ arc length $+ \frac{1}{4}$ circumference

$= 2r + r\theta + \frac{2\pi r}{4}$

$= 2r + r\theta + \frac{\pi r}{2}$ ✓

iv) ✓✓ NB 1ST PUT IN TERMS OF 1 VARIABLE using (ii)

NOT well done

a number of students did not recognise the calculus needed

$P = 2r + r\left(\frac{2}{r^2} - \frac{\pi}{2}\right) + \frac{\pi}{2}r$

$= 2r + \frac{2}{r} - \cancel{\frac{r\pi}{2}} + \cancel{\frac{\pi r}{2}}$

$= 2r + 2r^{-1}$

$P' = 2 - \frac{2}{r^2} = 0$

$2r^2 = 2$

$r = \pm 1$

but $r > 0 \therefore r = 1$

$P'' = +\frac{4}{r^3}$

$P''(1) > 0$ ∪ ∴ MIN when $r = 1$

v) ✓ $\theta = \frac{2}{1^2} - \frac{\pi}{2} = 0.429\ldots$

$\doteqdot 25^\circ$ nearest deg.

LOOK at parts (ii) & (iv) and note that this can be done even if nothing else has been done ☹ ///

b) $\frac{d}{dy}\, y\ln y - 7y \doteqdot \frac{d}{dy}\left((y\ln y - y) - 6y\right)$

✓✓ $= \ln y - 6$

using what is given

(OR use Product rule $y \times \frac{1}{y} + \ln y \times 1$.

c(i) ① $e^x = e^{6-x}$

$x = 6 - x$

$2x = 6$

$x = 3$

$y = e^3$ ✓

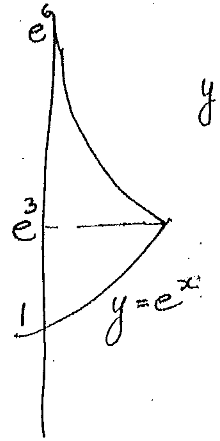
Many used long winded methods rather than seeing the simple index question this is!

OR

i) See below

ii)

(3)



$y = e^{6-x}$

$6-x = \ln y$

$x = 6 - \ln y$ ✓

$x = \ln y.$

✓

$$= \int_1^{e^3} \ln y \, dy + \int_{e^3}^{e^6} 6 - \ln y \, dy$$

$$= \Big[y\ln y - y\Big]_1^{e^3} + \Big[7y - y\ln y\Big]_{e^3}^{e^6}$$

$$= e^3 3 - e^3 - (1(0) - 1) + 7e^6 - e^6(6) - (7e^3 - e^3 3)$$

$$= 3e^3 - e^3 + 1 + 7e^6 - 6e^6 - 7e^3 + 3e^3$$

$$= 3e^3 - e^3 + 1 \underline{+ 7e^6 - 6e^6}$$

$$= \underline{\underline{e^6 - 2e^3 + 1}}$$ ✓

PTO FOR NOTE (364.76.)

NOTE although this gives correct answer it

$$A = \int_0^3 f(x) - g(x)\, dx = \int_0^3 e^{6-x} - e^{x}\, dx$$

$$= \left[\frac{e^{6-x}}{-1} - e^{x}\right]_0^3$$

$$= (-e^3 - e^3) - (-e^6 - e^0)$$

$$= -2e^3 + e^6 + 1$$

$$= \underline{e^6 - 2e^3 + 1}$$

did not gain full marks RTQ.!!

Too many lost marks for not reading the question

[12 marks]

2. (a) (i) $\frac{dV}{dt} = 330$ where V is in m^3 & t is in hours.

$V = \pi r^2 h = 0.0001\pi r^2 = \frac{\pi r^2}{10000}$ m^3

$A = \pi r^2$ and $V = 0.0001 A = \frac{A}{10000}$

$\therefore A = 10000V$

$\frac{dA}{dV} = 10000$

$\frac{dA}{dt} = \frac{dA}{dV} \times \frac{dV}{dt}$

$\frac{dA}{dt} = 10000 \times 330 = \underline{\underline{3{,}300{,}000 \ m^2/hr}}$ [2]

(ii) $\frac{dr}{dt} = 130$ Now $\frac{dA}{dr} = 2\pi r = \frac{dA}{dt} \times \frac{dt}{dr} = \frac{dA}{dt} \div \frac{dr}{dt}$

$2\pi r = \frac{3300000}{130}$ $\therefore r = 4040.087$ m

Hence $\underline{\underline{r = 4 \text{ km (nearest km)}}}$ [2]

(b) $\frac{\cos 2A}{\cos A} + \frac{\sin 2A}{\sin A} = \frac{(\cos^2 A - \sin^2 A)\sin A}{\cos A \sin A} + \frac{2\sin A \cos A \cos A}{\cos A \sin A}$

$= \frac{(2\cos^2 A - 1 + 2\cos^2 A)\cancel{\sin A}}{\cos A \cancel{\sin A}}$, $\sin A \neq 0$

$= \underline{\underline{\frac{4\cos^2 A - 1}{\cos A}}}$ as required. [3]

(c) (i) $-1 \leq 2x \leq 1$ $\therefore$ $\underline{\underline{-\frac{1}{2} \leq x \leq \frac{1}{2}}}$ (DOMAIN) [1]

(ii) $-\frac{\pi}{2} \leq \sin^{-1} 2x \leq \frac{\pi}{2}$ $\therefore$ $\underline{\underline{-\frac{3\pi}{2} \leq y \leq \frac{3\pi}{2}}}$ (RANGE) [1]

(d) (i) $\sum \alpha_i = \frac{-b}{a} = \frac{6}{2} = \underline{\underline{3}}$ [1]

(ii) $\sum \alpha_i \beta_i = \frac{c}{a} = \frac{4}{2} = \underline{\underline{2}}$ [1]

(iii) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma) = 3^2 - 2(2) = \underline{\underline{5}}$ [1]